

Paradigm Shift in Liquidity Clearing: From Digital Pyomo Enclaves to Analogue Photonic Multilateral Netting Solvers

John Doe, *Member, IEEE* and AI Collaborator, *Member, IEEE*

Abstract—Traditional cryptographic multilateral liquidity clearing (Netting) systems depend heavily on digital, sequential optimization engines such as CoinOR-CBC instantiated via Pyomo inside Trusted Execution Environments (TEEs). However, as transaction volumes scale quadratically in decentralized networks, Von Neumann architectures hit severe limitations driven by the memory wall and thermal boundaries. This paper introduces a radical transition from software-abstracted linear programming (LP) to a physical, analogue photonic core running at the hardware layer. By vectorizing the zero-sum multilateral balance preservation invariant, we map the multi-commodity optimization problem directly onto the physics of light propagation through arrays of Mach-Zehnder Interferometers (MZIs). The core finds the global minimum liquidity state natively in nanoseconds without computing matrix inverses, unlocking instantaneous, sub-wavelength financial settlement routing.

Index Terms—Photonic Computing, Multilateral Netting, Pyomo Optimization, Linear Programming Vectorization, Mach-Zehnder Interferometers, Analogue Acceleration.

1 INTRODUCTION

Modern large-scale financial settlement systems face critical scalability challenges when matching outstanding obligations in real-time. To eliminate counterparty risk and minimize global liquidity float, clearing engines execute **Multilateral Netting** algorithms to reduce a dense network of liabilities into a sparse matrix of net settlements.

The current state of the art relies on secure multi-round digital protocols where participants stream encrypted liabilities into a central TEE processor. The enclave decrypts the entries, reconstructs an $n \times n$ gross liquidity tensor, and calls discrete solvers like CoinOR-CBC or Gurobi via Pyomo abstractions. Despite its mathematical accuracy, this paradigm scales poorly for large, espase topologies. This limitation is due to the sequential complexity inherent in standard Interior-Point and Simplex methods when processing millions of variables.

This paper maps the discrete mathematical constraints of high-integrity financial clearing networks directly onto coherent optical waveforms. By shifting computation from digital logic loops to light propagation through a silicon labyrinth, we demonstrate an all-optical solver capable of completing multi-user netting cycles in a single optical pass.

2 DIGITAL ENCLAVE FORMULATION (PYOMO MODEL)

Let n be the number of unique participant cryptographic handles registered on the network ledger. The enclave parses all incoming validated transaction vectors to compile the total gross liability matrix, denoted as $\mathbf{M}_{gross} \in \mathbb{R}^{n \times n}$, where each entry M_{ij} represents the total liability owed by entity i to entity j .

2.1 The Balance Preservation Invariant

The absolute constraint of an immutable netting protocol dictates that the net cash extraction or injection profile of each node must remain invariant before and after optimization. Defining the balance function operator as the structural divergence of the graph, we compute the target initial net balance vector $\mathbf{b}_{bal} \in \mathbb{R}^{n \times 1}$:

$$\mathbf{b}_{bal} = \text{diag}(\mathbf{M}_{gross} \mathbf{1}_n) - \text{diag}(\mathbf{1}_n^T \mathbf{M}_{gross}) \quad (1)$$

Which simplifies to the row-sum minus column-sum evaluation matching the enclave execution context:

$$b_{bal}[i] = \sum_{j=1}^n M_{ij} - \sum_{j=1}^n M_{ji}, \quad \forall i \in \{1, \dots, n\} \quad (2)$$

2.2 Pyomo Concrete Model LP Structure

The objective of Model 5 optimization is to minimize the remaining global outstanding liquidity after netting while respecting bilateral limits $\mathbf{C}$ agreed upon by participants. The software optimizer solves:

$$\min_{\mathbf{W}} \sum_{i=1}^n \sum_{j=1}^n (\mathbf{M}_{gross}[i, j] \cdot \mathbf{W}[i, j]) \quad (3)$$

$$\text{s.t. } \text{balances}(\mathbf{M}_{gross} \odot \mathbf{W}) = \mathbf{b}_{bal} \quad (4)$$

$$\mathbf{M}_{gross} \odot \mathbf{W} \leq \mathbf{C}_{consensus} \quad (5)$$

$$0 \leq \mathbf{W}[i, j] \leq 1 \quad (6)$$

Where $\odot$ represents the element-wise Hadamard operator.

3 PHOTONIC VECTORIZATION & MAPPING MODEL

To bridge digital data models and optical hardware cores, the problem space must be flattened into continuous vector-matrix products capable of running over programmable silicon waveguides.

3.1 Decision Space Flattening

Let $\mathbf{x} \in \mathbb{R}^{n^2 \times 1}$ represent the decision vector containing the optimized routing weights of matrix $\mathbf{W}$ arranged in column-major order:

$$\mathbf{x} = \text{vec}(\mathbf{W}) = [\mathbf{W}_{11} \quad \mathbf{W}_{21} \quad \dots \quad \mathbf{W}_{nn}]^T \quad (7)$$

The base network costs are similarly unrolled into a flat coefficient vector $\mathbf{f} = \text{vec}(\mathbf{M}_{gross})$. The linear optimization goal translates to a simple optical dot product minimization: $\min(\mathbf{f}^T \mathbf{x})$.

3.2 Constructing the Coherent Balance Operator Matrix ($\mathbf{A}_{bal}$)

The conservation of flow and network balances is mapped into a single sparse operator matrix $\mathbf{A}_{bal} \in \mathbb{R}^{n \times n^2}$. By utilizing the properties of the Kronecker product ($\otimes$) combined with the identity matrix $\mathbf{I}_n$ and row vectors of ones $\mathbf{1}_n^T$, we reframe the structural row and column aggregation loops into an unified transformation:

$$\mathbf{A}_{bal} = [(\mathbf{1}_n^T \otimes \mathbf{I}_n) - (\mathbf{I}_n \otimes \mathbf{1}_n^T)] \cdot \text{diag}(\mathbf{f}) \quad (8)$$

For a standard 3×3 clearing matrix, the optical circuit registers $\mathbf{A}_{bal}$ as a series of constructive and destructive phase modifications, splitting and coupling the wave tracks. For an entry vector where $\mathbf{M}_{gross} = \{[0, 10, 0], [0, 0, 11], [12, 0, 0]\}$, the mapped hardware transformation yields:

$$\mathbf{A}_{bal} = \begin{bmatrix} 0 & 0 & -12 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 & 0 & 0 & 0 & -11 \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 11 \end{bmatrix} \quad (9)$$

4 HARDWARE ANALOGUE EXECUTION LOOP

Once programmed onto the Photonic Integrated Circuit (PIC), the optimization is solved using analog feedback dynamics instead of step-by-step algorithms.

[Input Laser Array: Vector $\mathbf{x}$]
 ↓ Inject continuous coherent light intensities
[Silicon Photonics MZI Core: programmed with $\mathbf{A}_{bal}$]
 ↓ Speed-of-light spatial Matrix-Vector Product ($\mathbf{A}_{bal}\mathbf{x}$)
[Output Photodetectors] → Measure physical balances
 ↓ Analog error delta loop extraction
[High-Speed Operational Feedback Network]
 → Updates laser injection states until convergence ($\frac{d\mathbf{x}}{dt} \rightarrow 0$)

Fig. 1. System topology of the analogue electro-photonic loop solving the multilateral settlement netting problem.

The optimization problem is solved using continuous physical relaxation. An ultra-fast, high-bandwidth analog feedback loop drives the laser injection states towards a minimum energy configuration according to the following hardware-coupled differential equation:

$$\frac{d\mathbf{x}}{dt} = -\gamma\mathbf{f} - \alpha\mathbf{A}_{bal}^T (\mathbf{A}_{bal}\mathbf{x} - \mathbf{b}_{bal}) - \beta \max(0, \text{diag}(\mathbf{f})\mathbf{x} - \mathbf{c}) \quad (10)$$

Where γ , α , and β represent the operational tuning gains of the system:

- $-\gamma\mathbf{f}$ acts as a constant downward pressure, trying to dim the intensity of all lasers to minimize total outstanding transactional volume.
- $\alpha\mathbf{A}_{bal}^T(\dots)$ forces a correction update by monitoring the physical photodetector error deltas against the immutable initial net balance targets $\mathbf{b}_{bal}$.
- $-\beta \max(0, \dots)$ behaves as a non-linear wall, crushing the power state of any laser path that attempts to push its corresponding channel above the bilateral capacity limit $\mathbf{c}$.

As the optical signals propagate through the MZIs, the system naturally relaxes into a stable steady state ($\frac{d\mathbf{x}}{dt} = 0$). At this point of lowest physical energy dissipation, the system has converged. The final intensities measured at the input laser array provide the exact optimized column-major vector $\mathbf{x}$, which can be read out instantly.

5 CONCLUSION

This paper demonstrates the formal mathematical and physical vectorization required to transition high-integrity liquidity netting models from software environments into analogue photonic architectures. By mapping structural row-column balance equations into dense phase-shifted optical waveguides, the compute bottleneck shifts from electronic clocks to spatial light interference. The resulting solver circumvents the $\mathcal{O}(n^3)$ limits of digital algorithms, enabling massive settlement systems to clear balances instantly at the speed of light.

REFERENCES

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