

Multilateral Set-Off Netting with Novation Constraints and Discretionary Credit Caps

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Abstract

This technical paper provides a step-by-step mathematical decomposition and architectural overview of the optimization core behind the Ecompensa 2.0 netting infrastructure. We present the exact linear programming formulation coded inside the secure execution runtime environment, highlighting how individual bilateral balances are compressed without introducing systemic risk or wealth distortion. Specifically, we analyze the mechanism of the structural novation credit channel. We demonstrate through an empirical test case how opening a dedicated credit limit rule ($\min(L_c, L_d)$) alters the mathematical boundaries of the network, dropping the required net settlement cash flow from \$30,000 to \$20,001.

1 Introduction

The primary objective of the Ecompensa 2.0 system is to address and resolve massive bilateral debt bottlenecks across small and medium-sized enterprise (SME) ecosystems. By executing synchronous multilateral clearing cycles, the system compresses a complex directed graph of raw outstanding liabilities into a sparse, hyper-efficient matrix of optimized final settlements.

The implementation acts strictly under legal frameworks (such as Article 12 of Decree-Law n.º 150/2019 in Portugal), ensuring that netting orders registered inside the network are irreversible and legally valid. The model analyzed in this paper focuses on the transition between pure commercial baseline set-off (Model 5: Zero-Added Counterparty Risk) and novation-based multi-user adjustments where participants intentionally open capacity buffers to unlock systemic liquidity.

2 Mathematical Optimization Framework

The core clearing problem is modeled as a continuous Linear Program (LP). The system minimizes the total aggregate residual debt lingering in the network post-clearing.

2.1 Graph Topology and Variables

Let $G = (V, E)$ be a directed network of commercial transactions, where V represents the set of registered corporate nodes ($|V| = r$), and E represents active corporate invoice relationships. The network is expanded to a complete system of $n = r \times r$ potential routes to map cross-party linkages seamlessly.

The principal continuous decision variable is defined as:

- $x_{i,j} \geq 0$: The optimized residual debt volume remaining active on the directed route from debtor j to creditor i after the optimization cycle has terminated.

2.2 The Objective Function

The objective function enforces maximum clearing efficiency by globally minimizing all remaining outstanding post-netting liabilities:

$$\min_x \sum_{i \in V} \sum_{j \in V} x_{i,j} \quad (1)$$

2.3 Fundamental Network Invariants and Constraints

To ensure structural safety, the linear programming solver is tightly bounded by four strict invariant sub-circuits:

1. **Conservation of Transactional Position (Zero Wealth Distortion):** The net position of each participant post-netting must be exactly identical to their net position pre-netting. This prevents any arbitrary or unbacked redistribution of capital:

$$\sum_{k \in V} x_{j,k} - \sum_{i \in V} x_{i,j} = \text{balancegross}_j \quad \forall j \in V \quad (2)$$

where $\text{balancegross}_j = \sum_{k \in V} \text{gross}_{j,k} - \sum_{i \in V} \text{gross}_{i,j}$.

2. **Credit Ceiling Invariant:** A participant cannot leave the clearing cycle holding a total credit exposure larger than their incoming commercial starting point:

$$\sum_{i \in V} x_{j,i} \leq \text{cregross}_j \quad \forall j \in V \quad (3)$$

3. **Debt Floor Invariant:** No participant can end the optimization round holding more total debt liabilities than they brought into the ecosystem:

$$\sum_{k \in V} x_{k,j} \leq \text{debgross}_j \quad \forall j \in V \quad (4)$$

4. **Bilateral Consensus Capacity Envelope:** Every optimized residual flow is strictly constrained by a collaborative boundary layer:

$$0 \leq x_{i,j} \leq L_{i,j} \quad \forall (i,j) \in E \quad (5)$$

3 The Mechanics of the Novation Credit Channel

In a pure commercial risk-isolated architecture, the bilateral capacities are restricted strictly to existing trade data ($L_{i,j} = \text{gross}_{i,j}$), meaning the system can only clear paths locked directly within commercial boundaries.

To expand clearing capabilities beyond rigid closed loops, Ecompensa 2.0 implements a ****Novation Credit Channel****. Under this regime, creditors propose an arbitrary discretionary limit line (L_c), and debtors symmetrically authorize a corresponding debit cap (L_d). The secure calculation perimeter establishes a consensus envelope using a localized logical lower-bound selection, which is then blended with the baseline commercial data:

$$L_{i,j} = \max(\min(L_c, L_d), \text{gross}_{i,j}) \quad (6)$$

This rule enables a flexible, risk-managed novation network. If both counter-parties agree to expand their limits, the optimization model can route intermediate liquidating transactions through this newly opened credit channel. This allows the network to bypass frozen links and tap into additional systemic liquidity.

4 Code Decomposition (Pyomo Implementation)

The mathematical properties detailed above reflect the production Pyomo Python script executed inside a Trusted Execution Environment (TEE). Below is the precise implementation structure:

```
1 import numpy as np
2 import pyomo.environ as pyo
3 from pyomo.environ import ConstraintList
4
5 # 1. Fetch data from Enclave memory
6 # gross represents the raw commercial input debt matrix
7 gross = np.asmatrix(gross_matrix, dtype=float)
8 r = gross.shape[0]
9 n = r * r
10
11 # 2. Establish the Novation Capacity Matrix Layer
12 consensus_limits_grid = np.zeros((r, r))
13 for c_pub in pub_list:
14     for d_pub in pub_list:
15         if c_pub == d_pub: continue
16         row = pub_to_idx[c_pub]
17         col = pub_to_idx[d_pub]
18         Lc = creditor_lines.get((c_pub, d_pub), 0)
19         Ld = debtor_lines.get((c_pub, d_pub), 0)
20
21         # Enforcing the Model 5 novation condition rule
22         consensus_limits_grid[row][col] = max(min(Lc, Ld), gross[row, col])
23
24 lim = np.asmatrix(consensus_limits_grid, dtype=float)
25
26 # 3. Preliminaries and Base Vectors
27 def balances(arr):
28     brr = arr - arr.transpose()
29     return np.matmul(brr, np.ones((arr.shape[0], 1)))
30
31 balancegross = balances(gross)
32 debgross = np.sum(gross, axis=0)
33 cregross = np.sum(gross, axis=1)
34
35 # 4. Instantiate Pyomo Concrete Model
36 model = pyo.ConcreteModel()
37 model.net = pyo.Var(range(n), bounds=(0, None))
38 net = model.net
39
40 # 5. Build Aggregation Expressions
41 cre_sum = []
42 for j in range(r):
43     cre_sum.append(sum([net[i + j * r] for i in range(r)]))
44
45 deb_sum = []
46 for j in range(r):
47     deb_sum.append(sum([net[i + j] for i in range(0, n, r)]))
48
49 model.balancesss = ConstraintList()
50 model.creditsss = ConstraintList()
51 model.debtsss = ConstraintList()
52
53 # 6. Bind Global System Invariants
54 for j in range(r):
55     net_bal = cre_sum[j] - deb_sum[j]
56     model.balancesss.add(expr=net_bal == balancegross[j, 0])
57     model.creditsss.add(expr=cre_sum[j] <= cregross[j, 0])
```

```

58     model.debtsss.add(expr=deb_sum[j] <= debgross[0, j])
59
60 # 7. Apply Flat Bilateral Consensus Capacities
61 limflat = lim.flatten().A1
62 model.limtt = ConstraintList()
63 for j in range(0, n):
64     model.limtt.add(expr=net[j] <= limflat[j])
65
66 # 8. Set Minimization Objective and Solve via CBC
67 net_sum = sum([net[i] for i in net])
68 model.obj = pyo.Objective(expr=net_sum, sense=pyo.minimize)
69 opt = pyo.SolverFactory('cbc')
70 results = opt.solve(model)

```

Listing 1: Core Pyomo Optimization Pipeline

5 Empirical Scenario Analysis: Decompressing the \$30,000 to \$20,001 Solution

To evaluate the mathematical impact of opening the novation credit channel, we observe two distinct states parsed through our runtime infrastructure under identical gross debt liabilities totaling **\$510,000**.

5.1 State A: Bounded Novation Optimization

In the first simulation round, the consensus capacities are constrained across a specific subset of links, limiting the solver’s routing flexibility. When evaluating the network topology, the constraint boundary prevents full structural ring-fencing.

- **Total Initial Gross Debt Claims:** \$510,000.00
- **Total Optimized Net Settlement Flow:** \$30,000.00
- **Absolute System Liquidity Savings:** \$480,000.00

The resulting concrete payment routes require a final transactional flow of \$30,000 to close the position matrices:

$$\begin{aligned}
 &\text{Handle 500100300} \xrightarrow{\text{pays } \$20,000.00} \text{Handle 500100100} \\
 &\text{Handle 500100100} \xrightarrow{\text{pays } \$10,000.00} \text{Handle 500100400} \\
 &\textbf{Total Cash Flow Required} = \textbf{\$30,000.00}
 \end{aligned}$$

5.2 State B: Extended Channel Limit Activation

In the second simulation round, a strategic adjustments vector is introduced. An extended novation channel constraint is modified across an active junction, injecting a microscopic fractional shift in baseline debt definitions (advancing the gross memory target to \$510,001 via an additional isolated \$1 component).

Crucially, opening this updated consensus boundary layer changes the feasible region of the linear programming problem. The expanded space allows the solver to find a more optimal path, clearing previously unresolvable dependencies.

- **Total Updated Gross Debt Claims:** \$510,001.00
- **Total Optimized Net Settlement Flow:** \$20,001.00

- **Absolute System Liquidity Savings:** \$490,000.00

The new clear-text solution maps the optimized payment routing as follows:

$$\begin{aligned} &\text{Handle 500100300} \xrightarrow{\text{pays \$10,001.00}} \text{Handle 500100400} \\ &\text{Handle 500100300} \xrightarrow{\text{pays \$10,000.00}} \text{Handle 500100100} \\ &\textbf{Total Cash Flow Required} = \textbf{\$20,001.00} \end{aligned}$$

5.3 Comparative Summary

By enabling participants to securely coordinate their mutual risk limits inside the TEE perimeter, the optimization engine successfully drives down the required liquid cash needed to clear the ecosystem.

Table 1: Impact of Novation Credit Channel Activation

System Metric	State A (Restricted)	State B (Channel Open)
Total Gross Network Liabilities	\$510,000.00	\$510,001.00
Required Net Settlement Flow	\$30,000.00	\$20,001.00
Absolute Liquidity Compressed	\$480,000.00	\$490,000.00
Net Settlement Efficiency	94.12%	96.08%

This optimization directly addresses the user’s objective: activating the collaborative channel limits shrinks the cash settlement requirement from \$30,000 to \$20,001. The solver shifts payments away from sequential multi-node pathways and condenses them into a highly optimized structure centered around Node 500100300. This modification frees up an extra \$10,000 in systemic liquidity without altering any participant’s net commercial balance.

6 Conclusion

The mathematical models analyzed in this paper confirm that multi-user debt netting performs optimally when commercial data is paired with discretionary credit channels. By utilizing Pyomo to structure these network constraints and deploying a CBC solver inside a secure TEE perimeter, Ecompensa 2.0 provides an efficient solution for corporate liquidity management. This approach eliminates intermediate transactional friction while strictly maintaining wealth preservation invariants.